Algorithms for Economically and Environmentally Efficient En Route Conflict Resolution

Georgia Tech | Air Transportation Laboratory

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Project Overview

- Congestion in the National Airspace System (NAS) are a source of unnecessary cost. $10billion
  - $6 billion impact upon direct airline operating costs
  - $4 billion impact upon the value of collective passenger time.
- Delays also have an environmental cost
  - Aircraft forced to fly far at sub-optimal cruise altitudes and/or speed
  - Sub-optimality fuel burn and gaseous emission that give rise to environmental concerns.
  - Current air traffic delays observed indicates that the air traffic control infrastructure is not capable of handing current traffic levels.
  - Forecast growth in aviation over the next decade there is an urgent need air traffic control decision-support or automation tools to address the problem of congestion in the NAS.
- We propose methods to investigate and quantify the economic and environmental benefits of enroute optimization tools.
  - Mathematical models for conflict-free optimal trajectories over a volume
  - Computational studies demonstrate savings due to proposed algorithms using traffic through Cleveland Air Route Traffic Control Center (ARTCC)

En Route Traffic Optimization

- Development of advanced air traffic conflict detection and resolution algorithms for overall improvement of the air traffic management (ATM) system.
  - Safety and capacity within the context of growing air traffic
  - Environmental implications.

Enroute Optimization

- Optimizing tactical control of aircraft to maintain separation while considering associated fuel costs
  - Safety requirements are considered as hard constraints
  - The objective function of the optimization program focuses on minimizing fuel costs

Optimization Framework

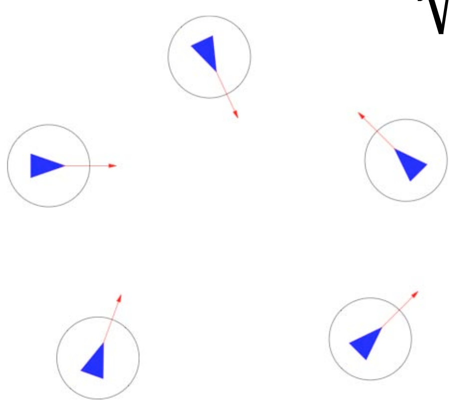
- The developed mathematical model is a mixed integer linear program,
  - Provides a broad framework for resolving conflicts through fast numerical optimization methods,
  - Allows for both heading and speed changes.
  - Particular focus has been placed on reducing fuel costs.
  - Ability to solve a complex problem in decision-time for conflicts involving a large number of aircraft.

Outline of Work

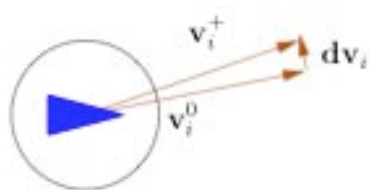
- Provide a general description of the problem
- Mathematical model for separation constraints required for conflict resolution
- Linear approximations of the complex fuel cost structures
- Computational studies
- Method for real-time implementation in a center environment
- Analysis.

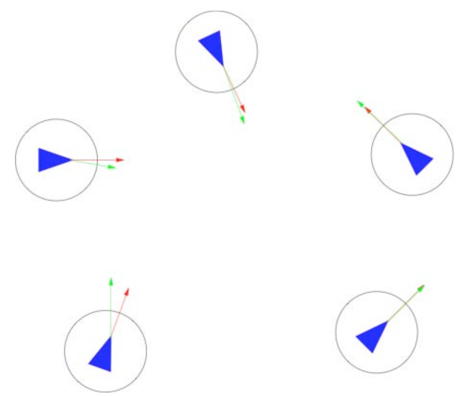
Problem Description

$$\sqrt{x_{dist}^2 + y_{dist}^2} \geq d_{i,j}^{\min} \ \forall \ t \ \varepsilon \ \Re^+$$



$$dv_i$$





$$\vec{p}_i = \begin{bmatrix} x_i & y_i \end{bmatrix}^T$$

$$\vec{v}_i = \begin{bmatrix} v_{i,x} & v_{i,y} \end{bmatrix}^T$$

$$x_{dist} = (x_i + v_{i,x}t) - (x_j + v_{j,x}t)$$

$$y_{dist} = (y_i + v_{i,y}t) - (y_j + v_{j,y}t)$$

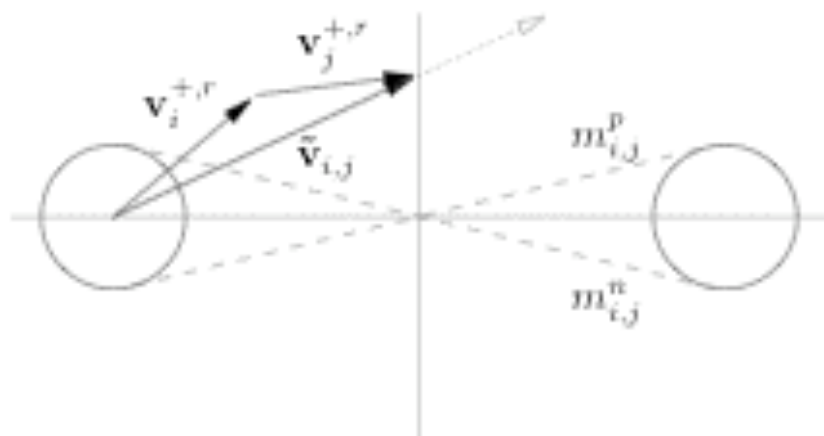
$$\{v_i^+\}$$

Separation Constraints

- Apply separation constraints to all pairs of aircraft
- Approach: Relative velocity between aircraft

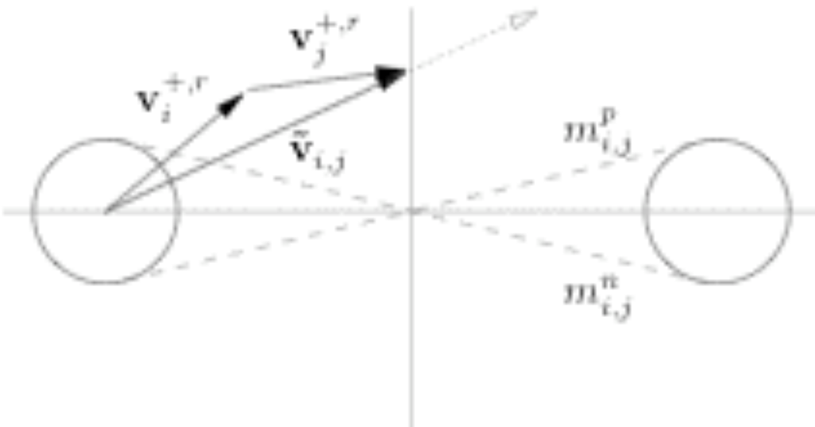
$$\frac{\tilde{v}_{i,j,y}}{\tilde{v}_{i,j,x}} \geq m_{i,j}^{p}$$

$$\tilde{v}_{i,j} = v_i^{+,r} - v_j^{+,r}$$



$$\frac{\tilde{v}_{i,j,y}}{\tilde{v}_{i,j,x}} \leq m_{i,j}^{n}$$

Separation Constraints



$$\frac{\tilde{v}_{i,j,y}}{\tilde{v}_{i,j,x}} \geq m_{i,j}^p$$

$$\frac{\tilde{v}_{i,j,y}}{\tilde{v}_{i,j,x}} \leq m_{i,j}^n$$

$$\tilde{v}_{i,j,y} \leq \tilde{v}_{i,j,x} m_{i,j}^n$$

$$\tilde{v}_{i,j,x} \geq 0$$

$$\tilde{v}_{i,j,y} \geq \tilde{v}_{i,j,x} m_{i,j}^p$$

$$\tilde{v}_{i,j,x} \geq 0$$

$$\tilde{v}_{i,j,x} \leq 0$$

Cost Formulation

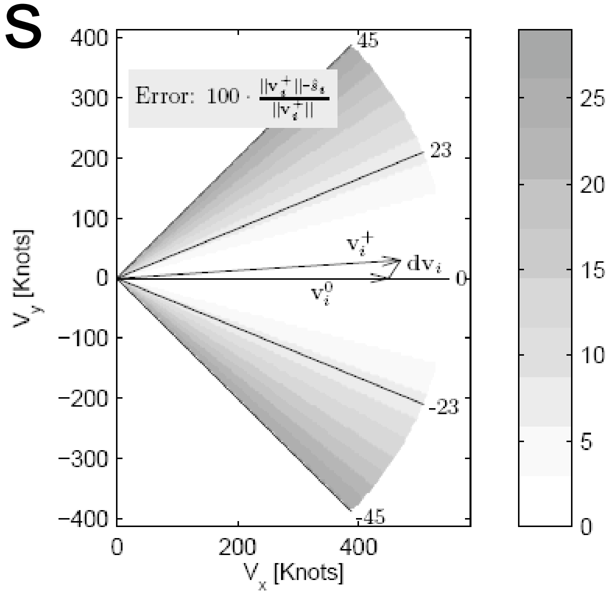
- In line with the primary goal of providing a framework in which fuel costs are considered in conflict resolution and aircraft routing

$$G_0(s,\Theta) = g_s(s) + g_h(\Theta)$$

Fuel Costs Due to Airspeed

- Previous research: $\min \|d\mathbf{v}\|_n$
- A measure of fuel costs requires tight approximations of airspeed

$$s_i^+ \sim s_i^0 + \frac{1}{s_i^0}\begin{bmatrix} v_{i,x}^0 & v_{i,y}^0 \end{bmatrix} dv_i = \hat{s}_i$$



Fuel Costs Due to Airspeed

- Development of tighter approximations through SOS2 variables

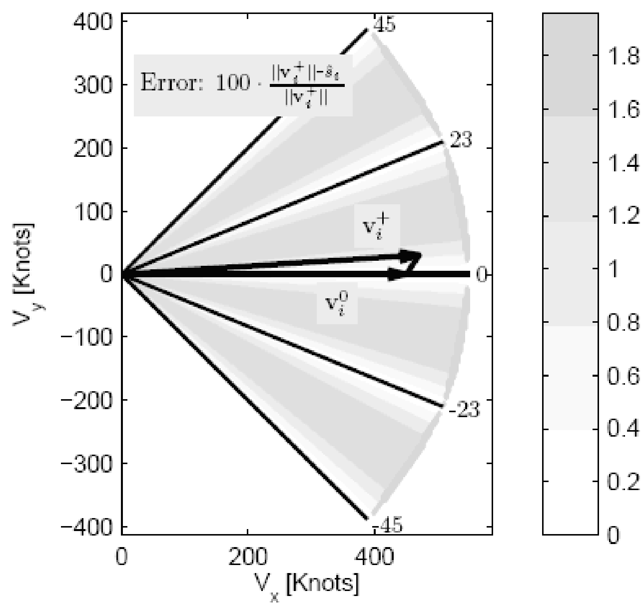
$$\hat{v}^{+}_{i,x} = \sum_{q=0}^{m} X_q \lambda_q$$

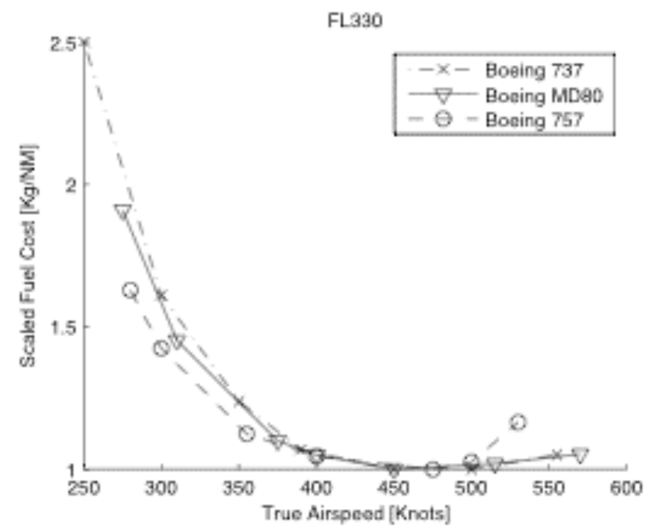
$$\hat{v}^{+}_{i,y} = \sum_{q=0}^{m} Y_q \lambda_q$$

$$\hat{s}_i = \sum_{q=0}^{m} Z_q \lambda_q$$

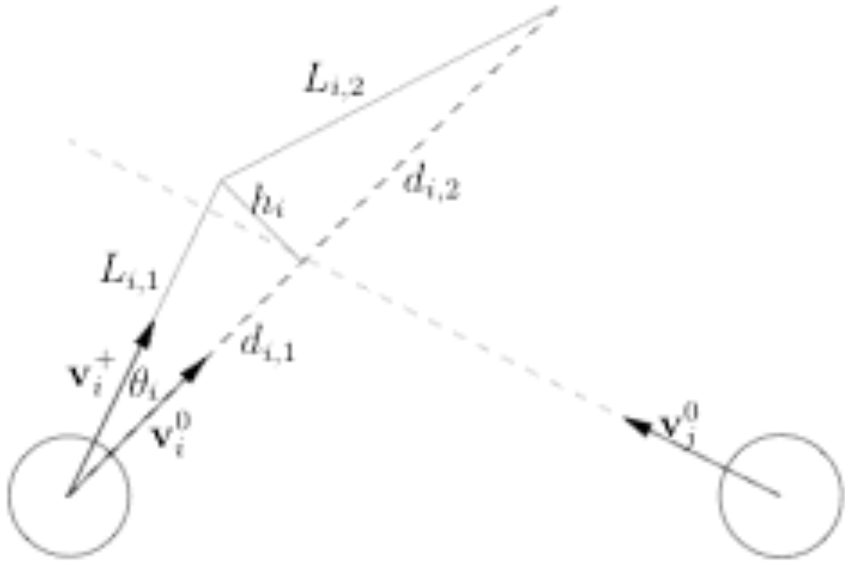
$$\sum_{q=0}^{m} \lambda_q = 1$$

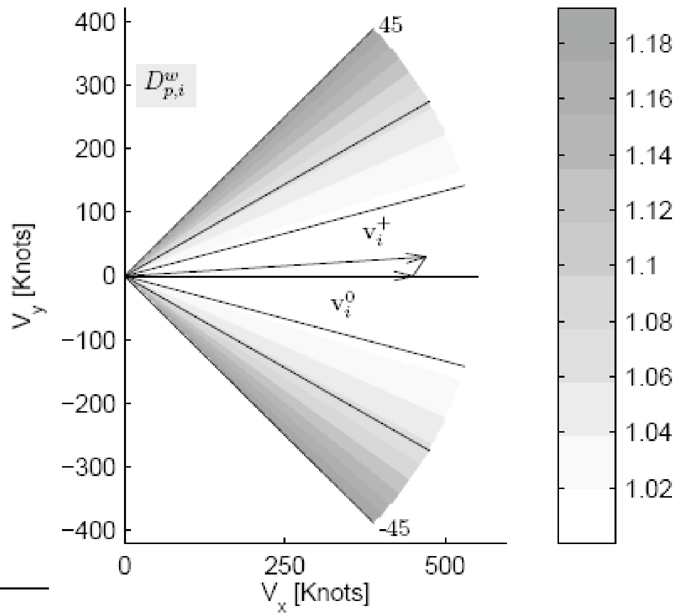
$$\lambda_q \ \varepsilon \ \text{SOS2} \ \forall \ q$$





Fuel Costs Due to Heading Changes





$$D_{p,i} = \frac{d_{i,1}/\cos(d\theta_i) + \sqrt{\left(d_{i,1}/\cos(d\theta_i)\right)^2 + D_i^2 - 2d_{i,1}D_i}}{D_i}$$

Fuel Costs Due to Heading Changes

$$x_w = v_i^{\max} \cos(\theta_w)$$

$$y_w = v_i^{\max} \sin(\theta_w)$$

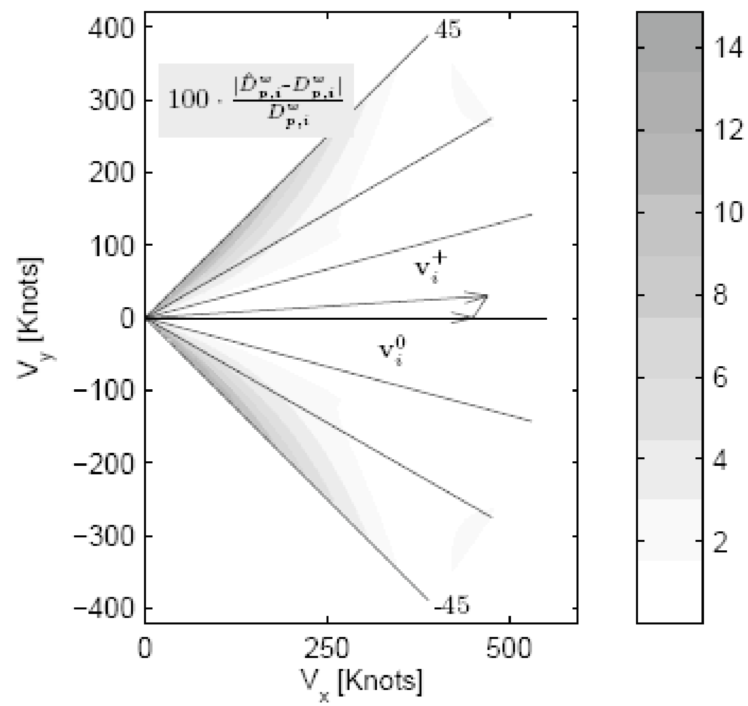
$$z_w = D_{p,i}(\theta_w)$$

$$x_{w+1} = v_i^{\max} \cos(\theta_{w+1})$$

$$y_{w+1} = v_i^{\max} \sin(\theta_{w+1})$$

$$z_{w+1} = D_{p,i}(\theta_{w+1})$$

$$\det\left(\begin{bmatrix} x & y & \hat{D}_{p,i}^{w} \\ x - x_w & y - y_w & \hat{D}_{p,i}^{w} - z_w \\ x - x_{w+1} & y - y_{w+1} & \hat{D}_{p,i}^{w} - z_{w+1} \end{bmatrix}\right) = 0$$



$$D_{p,i}^{w} = c_1 x + c_2 y + c_3$$

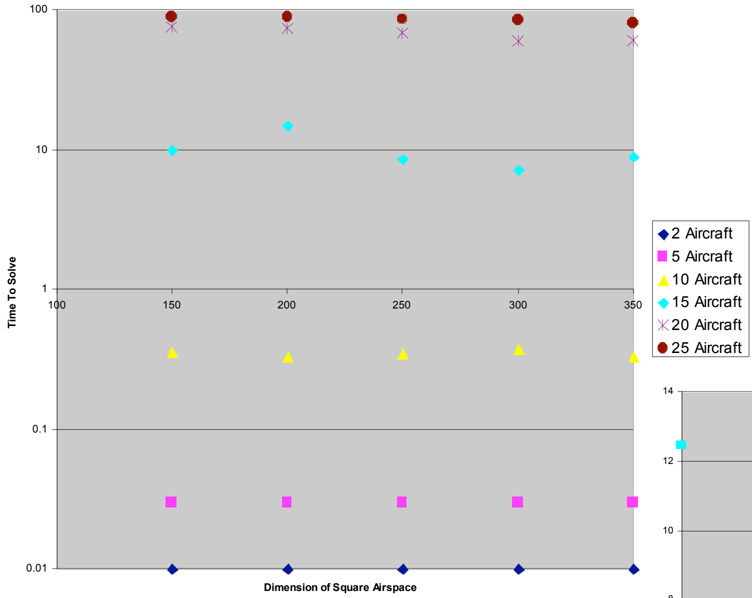
Model Review

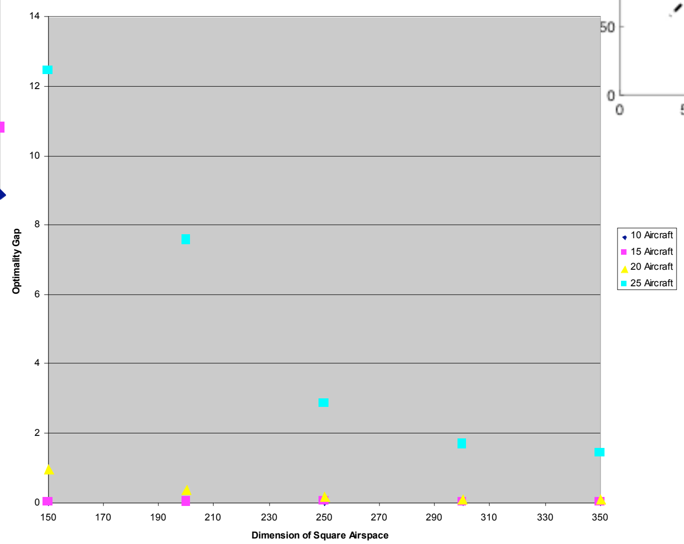
- Mixed Integer Linear Program
- Broad framework
  - Calculate Speed and Heading Changes
  - Any convex cost function
- Effectively solve in decision-time

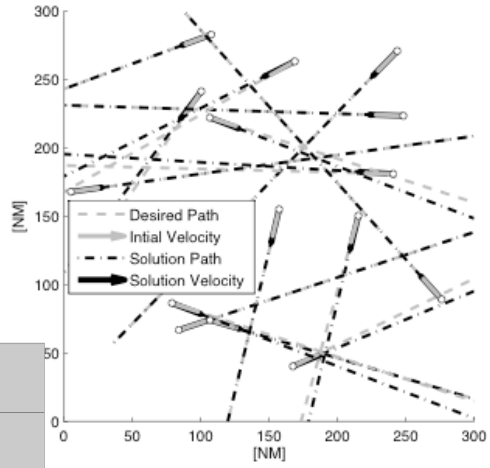
Computational Study

- Static Case
  - Stress test algorithms ability to solve difficult problems
- Dynamics Receding Horizon Case
  - Implementation of algorithm over “real-world” situation

Static Case


Time To Solve
Dimension of Square Airspace
2 Aircraft
5 Aircraft
10 Aircraft
15 Aircraft
20 Aircraft
25 Aircraft


Optimality Gap
Dimension of Square Airspace
10 Aircraft
15 Aircraft
20 Aircraft
25 Aircraft

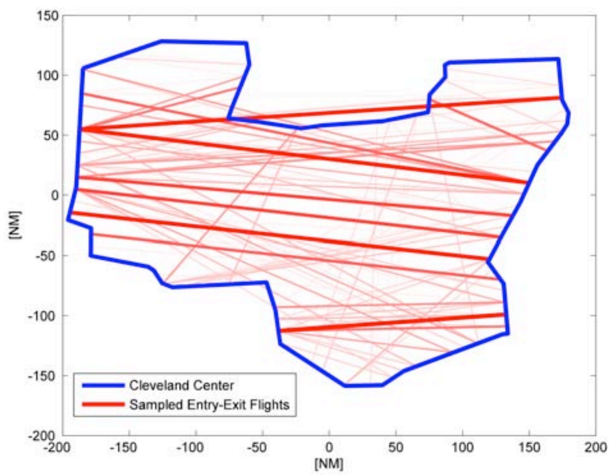

[NM]
[NM]
Desired Path
Intial Velocity
Solution Path
Solution Velocity

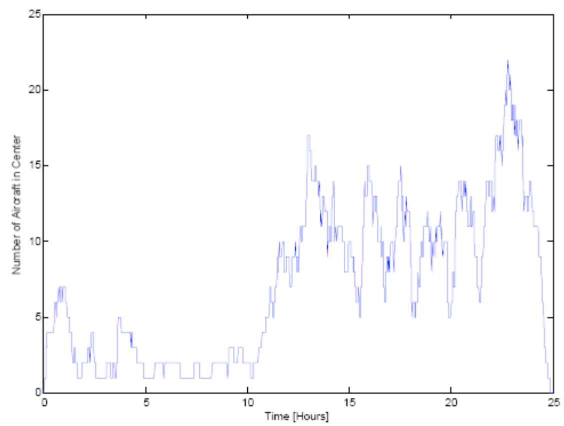
Receding Horizon Implementation

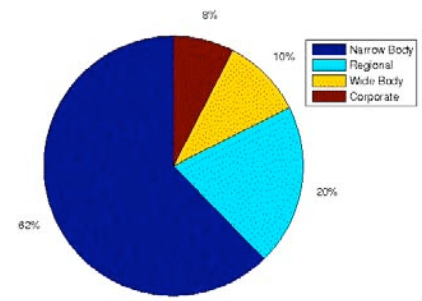
- Solving the conflict detection and resolution problem continuously in time.
  - At discrete intervals the problem is resolved as aircraft enter the air space
  - New velocity solutions are applied to aircraft.
  - Aircraft positions and velocities are propagated forward in time until the next time step
- Assuming that update times are greater than the computation time this method is feasible.

Case Study: Cleveland ARTCC

- 24hr Period - Sunday, May 1, 2005
  - FL36 (Westbound) – Nominal day in NAS

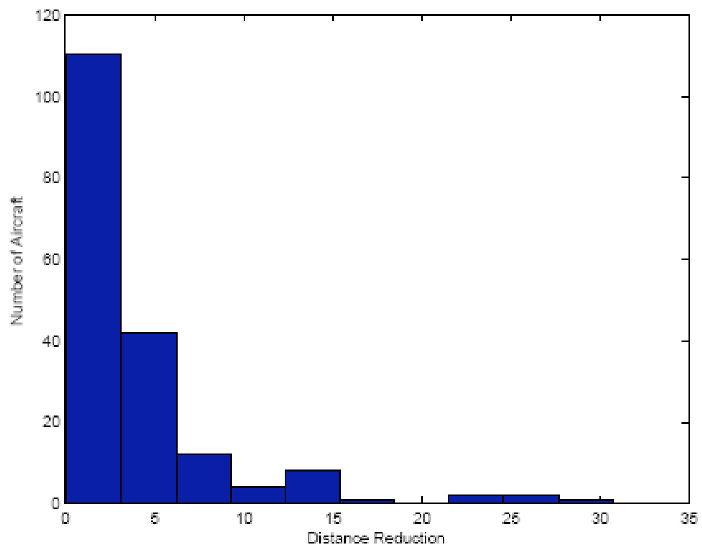


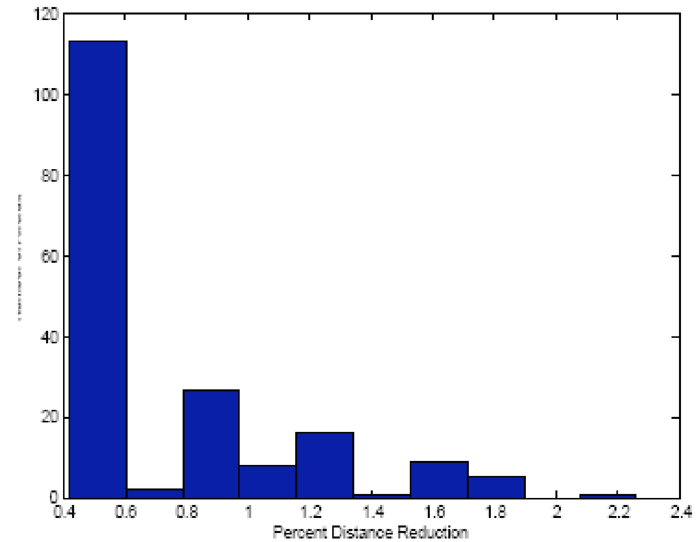




Case Study: Results

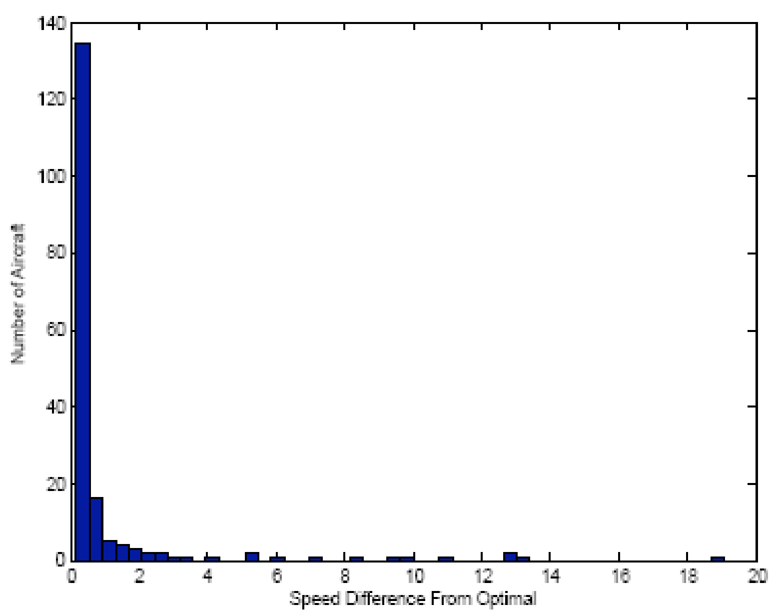
- Distance Traveled
  - Average Savings 9NM

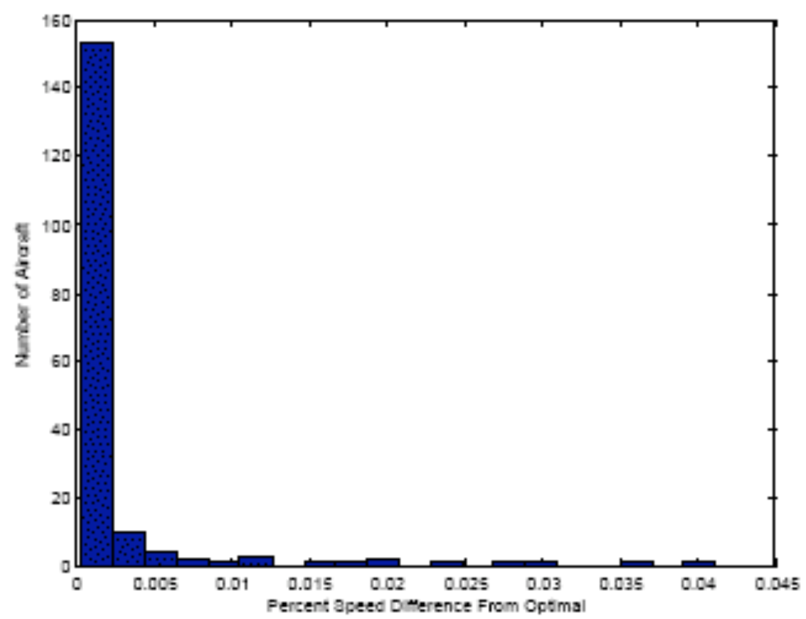




Case Study: Results

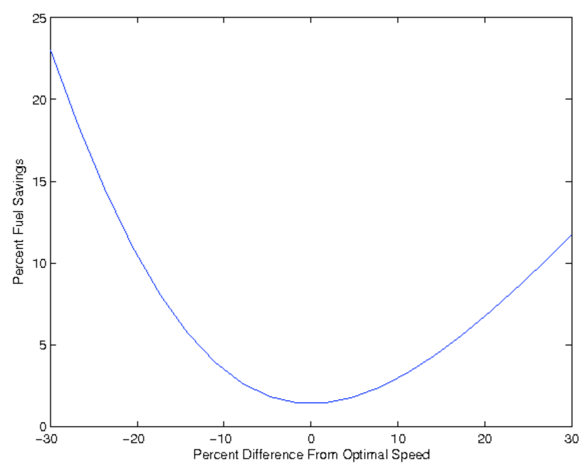
Aircraft Travel at/near Fuel Optimal Speeds





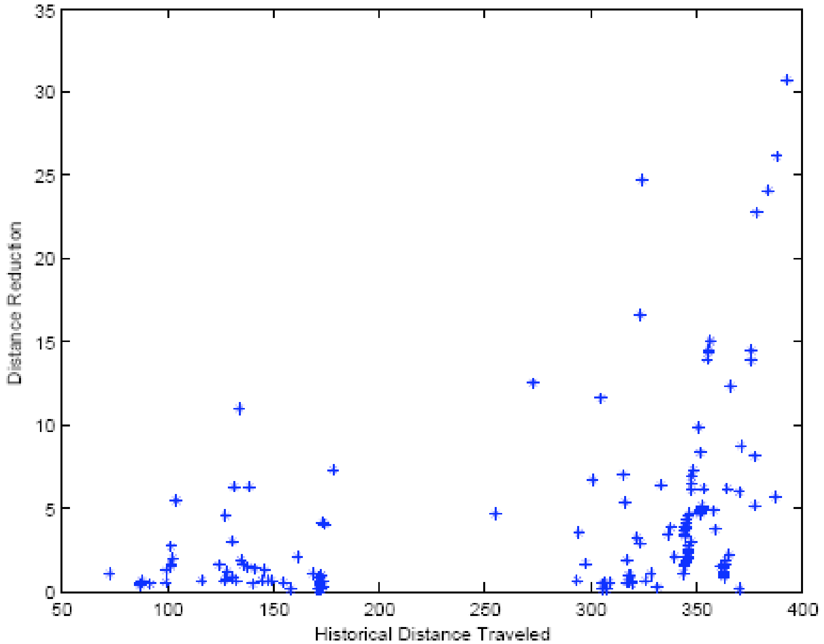
Case Study: Results

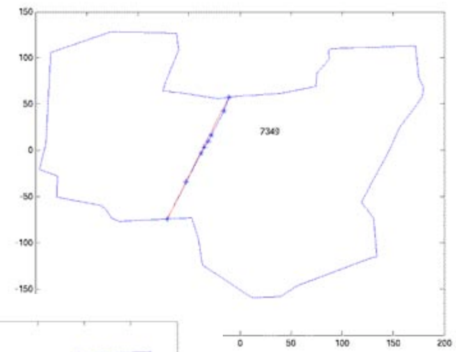
- Lower Bound on Fuel Savings: 1.4 %
- If aircraft fly at suboptimal speeds (0% or 15% below optimal): 3.37% and 6.13%
- Flights > 350NM, (24% of all flights): 2.1%.

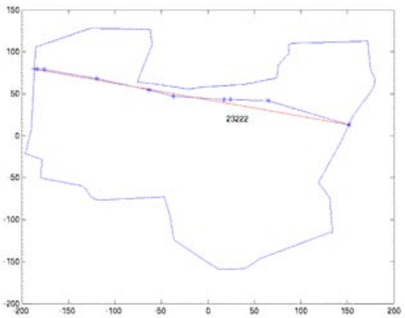


Case Study: Results

- The longer the flight, the greater benefit of fuel-optimal tactical control

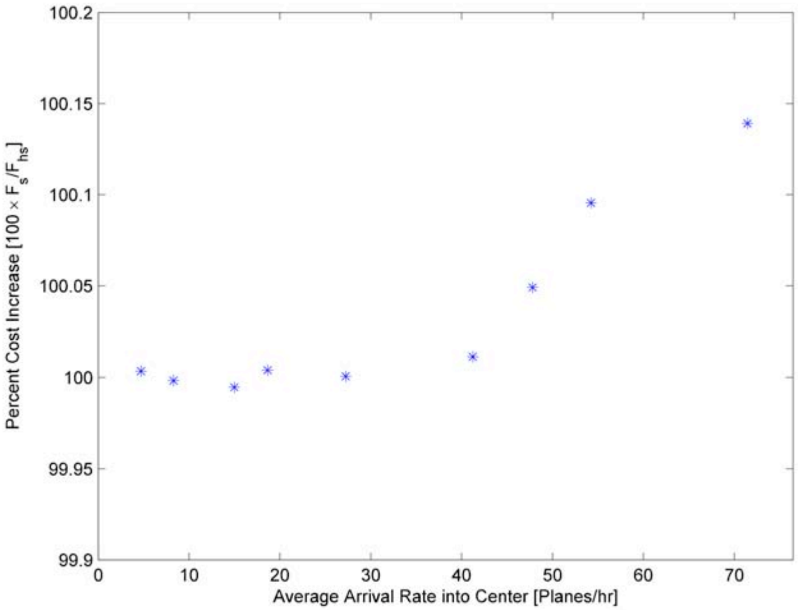






Increased Traffic Flow

- Proposed algorithm is increasingly effective as traffic flow increases



Weather (Movie)

Conclusions

- Optimal tactical control algorithm is an effective tool for conflict resolution.
- Decision-Time tools are possible.
- We can develop fuel optimal control schemes.